

Generation of True Random Numbers using quasi-Monte Carlo methods

Ana I Gomez, Domingo Gómez-Pérez, Florian Pausinger

Universidad de Cantabria, Queen's University Belfast

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TRNGs use physical processes to generate random numbers. Some characteristics

- Underlying physical process must be not possible to control
- Principle used to extract entropy and physical phenomenon limits the bit rate

Randomness source available from hardware accelerators (FPGAs, CPDs, ASICs).

Application to embedded security systems (cryptographic system-on-chip).
Methodology AIS 31

Challenges in modern embedded TRNG design

- i Finding a good quality source of randomness available in the digital technology
- ii Finding an efficient and robust principle of randomness extraction
- iii Guaranteeing the security (e.g. by a robust design or by an efficient online testing).

Mathematical assessment of the security: Lower bound of entropy per output bit

$$H(X) = -p \cdot \log_2(p) - (1 - p) \cdot \log_2(p),$$

where X is a random variable and p , bit probability

- Designs based on a stochastic model of the random process.
- If minimal entropy per output bit approaches 1, then the TRNG is an ideal RNG.

TRNG design by Cherkaoui et al. [1] that exploits the jitter noise of events propagating in a self-timed ring

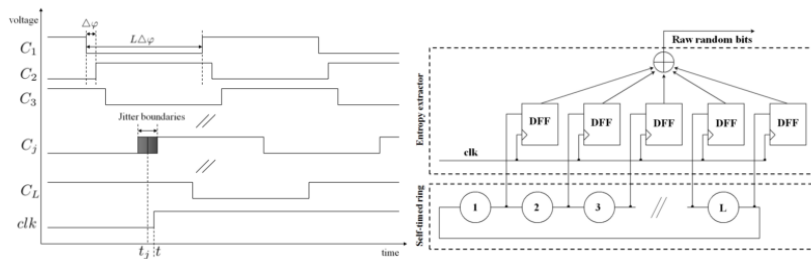
- Self-timed rings are oscillators in which several events can evolve evenly-spaced in time.
- Jitter noise make the significant instants of a digital signal vary from their ideal position in time.

The distance between two events is given by the phase difference

$$\Delta\phi = \frac{T}{2L}$$

where T , oscillation period. L , number of stages of the design.

TRNG Design



stochastic model

$\{C_i\}_{i=0}^{L-1}$: STR output signals, ordered by its mean time arrival t_i

$\{X_i\}_{i=0}^{L-1}$: random variable represents time position of each event,

probability distribution $N(t_i, \sigma^2)$

For a sampling time t , if $\sigma < \Delta\phi$

$$P[X_j < t] = \Phi\left(\frac{t - t_j}{\sigma}\right), P[X_{j-1} < t] = \Phi\left(\frac{t - t_j - \Delta\phi}{\sigma}\right)$$

Then, the probability that the output bit value at time t is equal to u is the probability of the XOR of the previous value

$$P_u(t) = 1 - \Phi\left(\frac{t - t_j}{\sigma}\right) - \Phi\left(\frac{t - t_j - \Delta\phi}{\sigma}\right) + 2\Phi\left(\frac{t - t_j}{\sigma}\right)\Phi\left(\frac{t - t_j - \Delta\phi}{\sigma}\right)$$

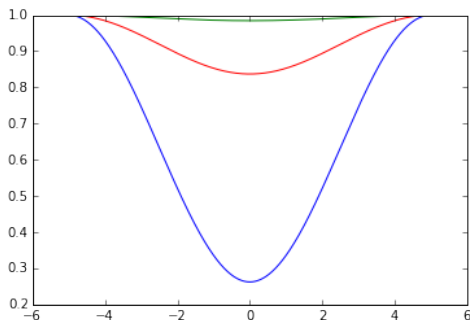


Figure: H versus t . $\sigma = \Delta\phi/4$ (Blue line), $\sigma = \Delta\phi/2$ (red line), corresponds to $\sigma = \Delta\phi$ (green line)

Non deterministic sampling time (jitter noise), t is the realization of a random variable $Z \sim N(t_s, \sigma_{\text{clk}}^2)$

$$P[X_j < Z] = \int_{-\infty}^{\infty} \Phi\left(\frac{t' - t_j}{\sigma}\right) \cdot f_z\left(\frac{t' - t}{\sigma_{\text{clk}}}\right) dt'$$

$$P[X_{j-1} < Z] = \int_{-\infty}^{\infty} \Phi\left(\frac{t' - t_j - \Delta\phi}{\sigma}\right) \cdot f_z\left(\frac{t' - t}{\sigma_{\text{clk}}}\right) dt'$$

$$P_u(t) = 1 - P[X_j < Z] - P[X_{j-1} < Z] + 2 \cdot P[X_{j-1} < Z] P[X_j < Z]$$

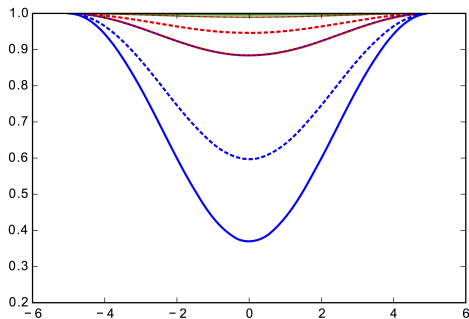


Figure: H versus t . $\sigma = \Delta\phi/4$ (Blue line), $\sigma = \Delta\phi/2$ (red line), corresponds to $\sigma = \Delta\phi$ (green line). Continuous line corresponds to $\sigma_{clk} = \sigma/2$, segmented line to $\sigma_{clk} = \sigma$, and dotted line corresponds to $\sigma_{clk} = 2\sigma$

Minimum entropy when n successive input bits are combined into one output bit for

- Random sampling points

Example: $\sigma = \frac{\Delta\phi}{4}$, $H \sim 1$ after 40 bits

- Uniform distributed sampling points

Example: $\sigma = \frac{\Delta\phi}{4}$, $H \sim 1$ after 20 bits when $f_{clock} = \frac{36}{125} \Delta\phi$

Finding an optimal set of sampling points, regarding:

- Optimal parameters for the underlying digital design
- Several STR with different frequencies



CHERKAOUI, A., FISCHER, V., AUBERT, A., & FRESQUET, L. *A Self-timed Ring Based True Random Number Generator*. In International symposium on advanced research in asynchronous circuits and systems- ASYNC 2013. (99–106), 2013.



CHERKAOUI, A., FISCHER, V., AUBERT, A., & FRESQUET, L. *A Very High Speed True Random Number Generator with Entropy Assesment*. In Cryptographic Hardware and Embedded Systems. CHES 2013. (179–196), 2013.

Thanks for your attention!