Monte Carlo method projective estimators for angular and temporal characteristics evaluation of polarized radiation

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Let us consider the following stationary integro-differential radiative transfer equation with polarization:

$$\omega\nabla\mathbf{\Phi}(r,\omega)+\sigma\mathbf{\Phi}(r,\omega)=\int\limits_{\Omega}\sigma_s P(\omega',\omega)\mathbf{\Phi}(r,\omega')\,d\omega'+\mathbf{f_0}(r,\omega),$$

or in the operator form

$$\mathbf{L}\mathit{\Phi}+\sigma\mathit{\Phi}=\mathbf{S}\mathit{\Phi}+\mathbf{f_0},$$

where $\mathbf{\Phi}=(\Phi_1,\Phi_2,\Phi_3,\Phi_4)^T$ – is a vector function describing the flux density of particles (of "vector photons"), or, in other words, the radiation intensity; Ω – is the space of unit direction vectors, $\omega\in\Omega$, $r\in D\subset\mathbb{R}^3$; $P(\omega',\omega)$ is the matrix scattering function, $\sigma=\sigma(r)$ is the total cross section, $\sigma=\sigma_s+\sigma_c$, σ_c is the absorption cross section, σ_s is the scattering cross section; $\mathbf{f_0}=(f_0^{(1)},f_0^{(2)},f_0^{(3)},f_0^{(4)})^T$ is the vector distribution density of particles.

The matrix $P(\omega', \omega)$ is defined by the relation

$$P(\omega', \omega) = L(\pi - i_2)R(\omega', \omega)L(-i_1),$$

where $L(\cdot)$ is a special rotation matrix

$$L(i) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2i & \sin 2i & 0 \\ 0 & -\sin 2i & \cos 2i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$R(\cdot)$ is the scattering matrix; i_1 is the angle between the plane ω', s and the scattering plane ω, ω'; i_2 is the angle between the scattering plane ω, ω' and the plane ω, s; $s = (0, 0, 1)$.

In the case of an isotropic medium, the scattering matrix is of the form:

$$R(\mu, r) = \frac{1}{2\pi} \begin{pmatrix} r_{11} & r_{12} & 0 & 0 \\ r_{21} & r_{22} & 0 & 0 \\ 0 & 0 & r_{33} & r_{34} \\ 0 & 0 & -r_{43} & r_{44} \end{pmatrix},$$

where $\mu = (\omega, \omega')$ is cosine of the scattering angle, r_{11} is the scattering function (or indicatrix), $\int_{-1}^{1} r_{11}(\mu)\, d\mu = 1$. If the scattering particles are homogeneous spheres, then $r_{11} = r_{22}$, $r_{12} = r_{21}$, $r_{33} = r_{44}$, $r_{34} = r_{43}$.

In the case of molecular (Rayleigh) scattering, the matrix R is defined as follows:

$$R(\mu) = \frac{1}{2\pi} \begin{pmatrix} 3(1+\mu^2)/8 & -3(1-\mu^2)/8 & 0 & 0 \\ -3(1-\mu^2)/8 & 3(1+\mu^2)/8 & 0 & 0 \\ 0 & 0 & 3\mu/4 & 0 \\ 0 & 0 & 0 & 3\mu/4 \end{pmatrix}.$$

Angular characteristics

Consider the radiation transport through a flat layer $0 < z < H$ of scattering and capturing media from the radiation source located on the border of $z = 0$ and directed in some direction ω_0.

Vector-function $\mathbf{\Phi}_\mathrm{s}(r,\omega)$ of the angular distribution of the scattered radiation flux at r point on the surface $z = h$, $0 \le h \le H$ is

$$\mathbf{\Phi}_\mathrm{s}(r,\mu,\varphi) = |\mu|\, \mathbf{\Phi}(r,\mu,\varphi),$$

where $\mu = (\omega, s)$, φ is the azimuthal angle.

Total radiation flux density (surface illumination) equal to

$$P_h = \int\limits_{\Omega} \Phi^{(1)}(r,\omega)\,\mu\, d\omega = \int\limits_0^{2\pi}\int\limits_0^{1} \Phi^{(1)}(r,\mu,\varphi)\,\mu\, d\mu d\varphi.$$

In the case of isotropic radiation, the total flux density is ${P_h}^{Lambert} = \pi\Phi^{(1)}(r)$ and the corresponding angular distribution of the radiation flux on a surface with a normalized density

$${\Phi_s^{(1)}}^{Lambert}(r,\mu,\varphi)/{P_h}^{Lambert} \equiv \mu/\pi, \;\; \mu \in (0,1),\; \varphi \in (0,2\pi)$$

is well-known Lambert distribution.

Study

- angular distribution of backscattered and transmitted radiation intensity

$$\Phi_s^{(1)}(x,y)/P_H,\ x \in (0,1), y \in (0,\pi),$$

- degree of polarization

$$p(r,\omega) = \frac{\sqrt{\Phi^{(2)}(r,\omega)^2 + \Phi^{(3)}(r,\omega)^2 + \Phi^{(4)}(r,\omega)^2}}{\Phi^{(1)}(r,\omega)}.$$

1: $\{\psi_i(x) \cdot \phi_j(y)\},\ i,j = 0,..\infty$, such that

$$\int_0^{2\pi}\int_0^1 x\,\psi_i(x)\phi_k(y)\psi_j(x)\phi_l(y)\ dx\ dy = \begin{cases} 1, & i=j, k=l \\ 0, & \text{else.} \end{cases}$$

$$\int_0^1 x\,\psi_i(x)\psi_j(x)\ dx = \begin{cases} 1, & i=j \\ 0, & i \neq j. \end{cases}$$

$$\int_0^{2\pi} \phi_k(y)\phi_l(y)\ dy = \begin{cases} 1, & k=l \\ 0, & k \neq l. \end{cases}$$

2: $\{H_{i,j}(x,y),\ i = 0,...\infty, j = -i,..,i\}$ such that

$$\int_0^{2\pi}\int_0^1 x\,H_{i,k}(x,y)H_{j,l}(x,y)\ dx\ dy = \begin{cases} 1, & i=j, k=l \\ 0, & \text{else.} \end{cases}$$

- N. V. Tracheva, S. A. Ukhinov (2017) *Numerical statistical study of the angular distribution of the polarized radiation scattered by medium//* Russian Journal of Numerical Analysis and Mathematical Modelling. Vol. 32(2), 135-146.

- Tracheva, N. V. (2017) *The use of two-dimensional projective Monte Carlo estimators for solution of number of problems of theory of radiation transfer//* Proceedings of the Conference of young scientists of Institute of computational mathematics and mathematical geophysics of SB RAN. Novosibirsk, 52–63.

Functions $\psi_i(x)$ were obtained from Jacobi polynomials:

$$\psi_i(x) = \sqrt{2i+2}\sum_{k=0}^{i} \frac{(-1)^k\,(2\,i+1-k)!}{(i-k)!\,k!\,(i+1-k)!}\,x^{i-k}.$$

Functions $\phi_j(x)$ were obtained from Legendre polynomials:

$$\phi_j(y) = \frac{\sqrt{2j+1}\,(-1)^j}{\sqrt{2\pi}}\sum_{k=0}^{j} \frac{(j+k)!}{(j-k)!\,k!\,k!\,(2\pi)^k}\,(-x)^k.$$

- Mahotkin O. A. (1996) *Analysis of radiative transfer between surfaces by hemispherical harmonics*//Journal of Quantitative Spectroscopy and Radiative Transfer. Vol. 56(6), 869-879.

The basis is constructed on the basis of hemispherical functions $\{H_{i,j}(\mu,\varphi)\}$ of a form

$$H_{0,0}(\mu,\varphi) = \frac{1}{\sqrt{\pi}}; \qquad H_{i,0}(\mu,\varphi) = \frac{\sqrt{i+1}}{\sqrt{\pi}} \sum_{k=0}^{i} \frac{(-1)^k\,(2\,i+1-k)!}{(i-k)!\,k!\,(i+1-k)!}\,\mu^{i-k},\ i=1,2,...;$$

$$H_{i,j}(\mu,\varphi) = \frac{\sqrt{2(i+1)}}{2^j\sqrt{\pi\cdot\prod_{k=1}^{j}[(i+1)^2-k^2]}}\cdot\cos(j\varphi)\times$$

$$\times\sum_{k=0}^{i-j}\frac{i!(i+j+k+1)!(i+j+1)!(\mu-1)^k(1-(2\mu-1)^2)^{j/2}}{k!(l+j+1)!(j+k)!(i-j-k)!(i+1)!},\ i=1,2,...,j=1,...,i;$$

$$H_{i,-j}(\mu,\varphi) = \frac{\sqrt{2(i+1)}}{2^j\sqrt{\pi\cdot\prod_{k=1}^{j}[(i+1)^2-k^2]}}\cdot\sin(j\varphi)\times$$

$$\times\sum_{k=0}^{i-j}\frac{i!(i+j+k+1)!(i+j+1)!(\mu-1)^k(1-(2\mu-1)^2)^{j/2}}{k!(l+j+1)!(j+k)!(i-j-k)!(i+1)!},\ i=1,2,...,j=1,...,i.$$

Vector-function of the angular distribution of the scattered radiation flux on the surface $z = h$, $0 \le h \le H$:

$$\boldsymbol{\Phi}_s(x,y) = x \sum_{i=0}^{\infty} \sum_{j=-i}^{i} \boldsymbol{a}_{i,j} H_{i,j}(x,y),$$

where

$$\boldsymbol{a}_{i,j} = \int_0^{2\pi} \int_0^{1} \boldsymbol{\Phi}_s(x,y) H_{i,j}(x,y) \, dx \, dy.$$

$$\boldsymbol{\xi}_{i,j} = \sum_{k=1}^{N_t-1} q_k \mathbf{Q}_{k+1} H_{i,j}(\mu_{k+1}, \varphi_{k+1}) e^{-\dfrac{\sigma(h-z_k)}{\mu_{k+1}}} \Delta_{s,\mu}(z_k, \mu_{k+1}), \quad \mathrm{E}\boldsymbol{\xi}_{i,j} = a_{i,j}.$$

$$\xi_h = \sum_{k=1}^{N_t-1} q_k \mathbf{Q}_{k+1} e^{-\dfrac{\sigma(h-z_k)}{\mu_{k+1}}} \Delta_{s,\mu}(z_k, \mu_{k+1}), \quad \mathrm{E}\xi_h = P_h.$$

constructed on a Markov chain $\{x_k\}$, where $x_k = (z_k, \omega_k)$, z_k – z-coordinate of the collision point, $z_0 = 0$; ω_k – the direction of the particle before the collision event, $\omega_1 = \omega_0$; $\mu_k = (\omega_k, \mathbf{e}_z)$;

$$\Delta_{s,\mu}(z_k, \mu_{k+1}) = \begin{cases} 1, & \text{if } z_k < h \text{ and } \mu_{k+1} > 0 \\ 0, & \text{else,} \end{cases}$$

i.e. $\Delta_{s,\mu}$ – the indicator of the surface $z = h$ intersection by the particle direction after the scattering event in the point $z_k < h$, N_t – random number of the last collision event. Direction of the scattering ω_k in point z_{k-1} simulated according to the element R_{11} of the scattering matrix $R(\omega_{k-1}, \omega_k)$. Vector weight $\mathbf{Q}_k = \left(Q_k^{(1)}, Q_k^{(2)}, Q_k^{(3)}, Q_k^{(4)}\right)^T$ defines by:

$$\mathbf{Q}_1 = \left(I_0^{(1)}, I_0^{(2)}, I_0^{(3)}, I_0^{(4)}\right)^T,$$

$$\mathbf{Q}_k = P(\omega_{k-1}, \omega_k)\mathbf{Q}_{k-1}/R_{11}(\omega_{k-1}, \omega_k), \quad k \geq 2.$$

$$\mathbf{\Phi}(x,y) \approx \sum_{i=0}^{n} \sum_{j=-i}^{i} \boldsymbol{a}_{i,j} H_{i,j}(x,y) = \mathbf{\Phi}_n(x,y) \approx \tilde{\mathbf{\Phi}}_n(x,y) = \sum_{i=0}^{n} \sum_{j=-i}^{i} \boldsymbol{\alpha}_{i,j} H_{i,j}(x,y),$$

where

$$\boldsymbol{\alpha}_{i,j} = \mathrm{E}_N \boldsymbol{\xi}_{i,j} \equiv \frac{1}{N} \sum_{m=0}^{N} \boldsymbol{\xi}_{i,j}^{(m)},$$

$\boldsymbol{\xi}_{i,j}^{(m)}$ – random value realization $\boldsymbol{\xi}_{i,j}$ on m-th trajectory,

$$\mathrm{E}\boldsymbol{\alpha}_{i,j} = \mathrm{E}\boldsymbol{\xi}_{i,j} = \boldsymbol{a}_{i,j},$$

$$\mathrm{E}\tilde{\mathbf{\Phi}}_n(x,y) = \mathbf{\Phi}_n(x,y).$$

Componentwise representation of variation of random vector-function $\tilde{\mathbf{\Phi}}_n(x)$

$$D\tilde{\mathbf{\Phi}}_n(x,y) = \sum_{i=0}^{n}\sum_{k=-i}^{i}\sum_{j=0}^{n}\sum_{l=-j}^{j} \mathrm{cov}(\boldsymbol{\alpha_{i,k}}, \boldsymbol{\alpha_{j,l}}) H_{i,k}(x,y) H_{j,l}(x,y) =$$

$$\frac{1}{N}\sum_{i=0}^{n}\sum_{k=-i}^{i}\sum_{j=0}^{n}\sum_{l=-j}^{j} \mathrm{cov}(\boldsymbol{\xi_{i,k}}, \boldsymbol{\xi_{j,l}}) H_{i,k}(x,y) H_{j,l}(x,y).$$

$$\tilde{p}_n(x,y) = \frac{\sqrt{(\tilde{\Phi}_n^{(2)}(x,y))^2 + (\tilde{\Phi}_n^{(3)}(x,y))^2 + (\tilde{\Phi}_n^{(4)}(x,y))^2}}{\tilde{\Phi}_n^{(1)}(x,y)}.$$

$$\mathrm{D}_N \tilde{p}_n(x) = \frac{1}{g_1^2}\Bigg(\frac{g\,\mathrm{D}_N g_1}{g_1^2} + \frac{g_2^2\,\mathrm{D}_N g_2 + g_3^2\,\mathrm{D}_N g_3 + g_4^2\,\mathrm{D}_N g_4}{g} - $$
$$- 2\frac{\mathrm{cov}_N(g_1, g_2)\,g_2 + \mathrm{cov}_N(g_1, g_3)\,g_3 + \mathrm{cov}_N(g_1, g_4)\,g_4}{g_1} +$$
$$+ 2\frac{\mathrm{cov}_N(g_2, g_3)\,g_2 g_3 + \mathrm{cov}_N(g_2, g_4)\,g_2 g_4 + \mathrm{cov}_N(g_3, g_4)\,g_3 g_4}{g}\Bigg) +$$
$$+ o(N^{-1}),$$

where $g_m = \tilde{\Phi}_n^{(m)}(x, y),\ g = g_2^2 + g_3^2 + g_4^2,$

$$\mathrm{cov}_N(g_m, g_t) = \frac{1}{N}\sum_{i=0}^{n}\sum_{k=-i}^{i}\sum_{j=0}^{n}\sum_{l=-j}^{j} \mathrm{cov}_N(\xi_{i,k}^{(m)}, \xi_{j,l}^{(t)})\, H_{i,k}(x, y) H_{j,l}(x, y), \quad m, t = 1, ..., 4.$$

Temporal characteristics

An exponential asymptotics parameter is the principal eigenvalue λ^* of the homogeneous stationary transfer equation:

$$\mathbf{L}\mathbf{\Phi} + (\sigma + \lambda/v)\mathbf{\Phi} = \mathbf{S}\mathbf{\Phi},$$

with standard boundary conditions (see Davison B. *Neutron Transport Theory*. Clarendon, Oxford. 1957.).

In the work of Mikhailov G. A, Tracheva N. V., Uhinov S. A. (Monte Carlo study of time asymptotics of the polarized radiation intensity // *Computational Mathematics and Mathematical Physics. 2007. V. 47, N. 7.*) this statement was extended to polarized radiation. In the spatially homogeneous case (i.e., when the entire space is filled with a homogeneous medium), this goal can be achieved rather easily through weighted simulation of radiative transfer. Moreover, it proves that $\lambda^* = \lambda^*_\infty = -\sigma_c v$ irrespective of the polarization type. Here, σ_c is the absorption cross section and v is the velocity of the particles.

It is well known that $\lambda^* = \lambda^*_\infty = -\sigma_c v$ for a half-space as well.

A number of publications, e.g.

- Romanova L. M. (1965) Limit cases of the free path distribution function of photons exiting a thick light-diffusing layer.// *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana.*, **1**, No. 6, 599 - 606.

- Zege E.P., Katsev I.L. (1973) *Time asymptotical solutions of radiation transfer equation and their application.* Preprint. Inst. Fiz. Akad. Nauk. BSSR, Minsk.

show that for intensity functionals without polarization the following asymptotic relation exists:

$$J(r,\omega,t) \sim C(r,\omega) \cdot t^{\alpha} \cdot e^{-\sigma_c v t}, \qquad t \to +\infty.$$

We are concerned with obtaining of the similar relation in pre-asymptotical times.

Let us consider a system $\psi_i(t)$ of orthonormal over $(0, \infty)$ with weight $e^{-\sigma_c v t}$ functions, such that

$$\int\limits_0^\infty e^{-\sigma_c v t}\,\psi_i(t)\psi_j(t)\ dt = \begin{cases} 1, & i = j, \\ 0, & \text{else.} \end{cases}$$

$$\mathbf{\Phi}(r, \omega, t) = C(r, \omega) e^{-\sigma_c v t} \sum_{i=0}^{\infty} a_i \psi_i(t),$$

where

$$a_i = C(r, \omega) \int\limits_0^\infty \mathbf{\Phi}(t) \psi_i(t)\, dt.$$

$$\boldsymbol{\xi}_i = \sum_{k=1}^{N_t-1} q_k \mathbf{Q}_{k+1} \psi_i(t_{k+1}), \quad E\xi_i = a_i$$

built on Markov chain $\{x_k\}$, $x_k = (r_k, \omega_k)$, r_k – coordinates of the collision point, $r_0 = (0,0,0)$; ω_k – is the direction of the particle before the collision, $\omega_1 = \omega_0$; $\mu_k = (\omega_k, \mathbf{e}_z)$, N_t – random number of the last collision event, the scattering direction ω_k at the point r_{k-1} is simulated according to an element R_{11} of scattering matrix $R(\omega_{k-1}, \omega_k)$. Vector weigt $\mathbf{Q}_k = \left(Q_k^{(1)}, Q_k^{(2)}, Q_k^{(3)}, Q_k^{(4)}\right)^T$ is determined by

$$\mathbf{Q}_1 = \left(I_0^{(1)}, I_0^{(2)}, I_0^{(3)}, I_0^{(4)}\right)^T,$$

$$\mathbf{Q}_k = P(\omega_{k-1}, \omega_k)\mathbf{Q}_{k-1}/R_{11}(\omega_{k-1}, \omega_k), \quad k \geq 2.$$

$$\mathbf{\Phi}(r,\omega,t) \approx C(r,\omega)e^{-\sigma_c vt}\sum_{i=0}^{n} a_i\psi_i(t) = \mathbf{\Phi}_n(r,\omega,t) \approx$$

$$\approx \tilde{\mathbf{\Phi}}_n(r,\omega,t) = C(r,\omega)e^{-\sigma_c vt}\sum_{i=0}^{n} \alpha_i\psi_i(t),$$

where

$$\alpha_i = \mathrm{E}_N\xi_i \equiv \frac{1}{N}\sum_{m=0}^{N} \xi_i^{(m)}\,,$$

$\xi_i^{(m)}$ – random value ξ_i realization on m-th trajectory,

$$\mathrm{E}\alpha_i = \mathrm{E}\xi_i = a_i,$$

$$\mathrm{E}\tilde{\mathbf{\Phi}}_n(r,\omega,t) = \mathbf{\Phi}_n(r,\omega,t).$$

Variation of random function $\tilde{\mathbf{\Phi}}_n(t)$ is equal to

$$D\tilde{\mathbf{\Phi}}_n(t) = C^2(r,\omega)e^{-2\sigma_c vt}\sum_{i=0}^{n}\sum_{j=0}^{n}\mathrm{cov}(\boldsymbol{\alpha_i},\boldsymbol{\alpha_j})\psi_i(t)\psi_j(t) =$$

$$= C(r,\omega)^2 e^{-2\sigma_c vt}\frac{1}{N}\sum_{i=0}^{n}\sum_{j=0}^{n}\mathrm{cov}(\boldsymbol{\xi_i},\boldsymbol{\xi_j})\psi_i(t)\psi_j(t).$$

As orthonormal basis let us consider a modyfied Laguerre polynomials $L_n(x)$, orthogonal with weight e^{-x} over $(0, \infty)$. We can obtain the following explicit form for functions $\psi_i(t)$:

$$\psi_i(x) = \sqrt{\sigma_c v} \sum_{k=0}^{i} \frac{(-1)^k \, i!}{k! \, (i-k)! \, k!} \, {\sigma_c}^k \, v^k \, t^k.$$

Numerical results

- Deirmendjian, D. (1969) *Electromagnetic Scattering on Spherical Polydispersions. Amer. Elsevier, New York.*

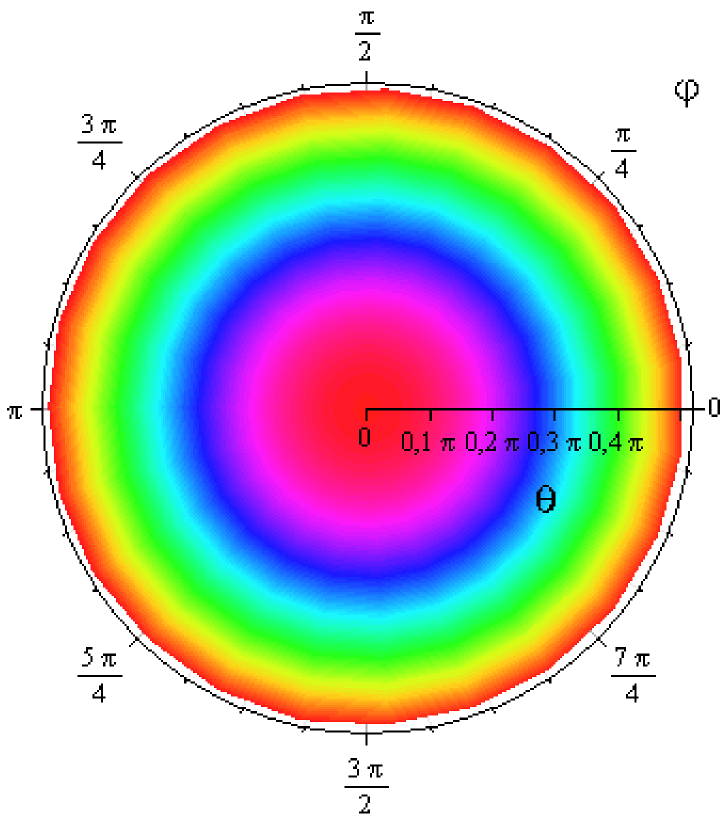
Aerosol scattering.
Coefficient of refraction on particles $n = 1.331 - i1.310^{-4}$ (water), particle size distribution is logarithmically normal with density $f(r) = \frac{1}{r}\exp(-\frac{1}{2\sigma_g^2}\ln^2(\frac{r}{r_g})), r \in (0, 10mkm), r_g = 0.12mkm, \sigma_g = 0.5$, wave-length of radiation equals to 0.65 mkm. Mean cosine of scattering angle - $\mu_0 = 0.7292$.

- Marchuk, G. I., Mikhailov, G. A., Nazaraliev, M. A. et al. (1980) *The Monte Carlo Methods in Atmospheric Optics. Amer. Springer-Verlag, Heidelberg .*

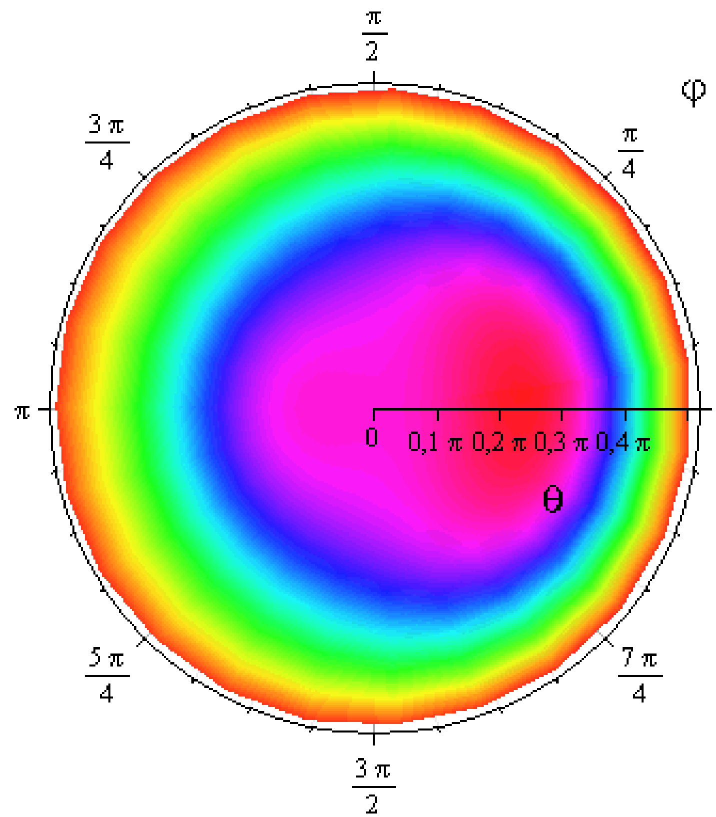
Molecular scattering.

$$R(\mu) = \frac{1}{2\pi}\begin{pmatrix} 3(1+\mu^2)/8 & -3(1-\mu^2)/8 & 0 & 0 \\ -3(1-\mu^2)/8 & 3(1+\mu^2)/8 & 0 & 0 \\ 0 & 0 & 3\mu/4 & 0 \\ 0 & 0 & 0 & 3\mu/4 \end{pmatrix}.$$

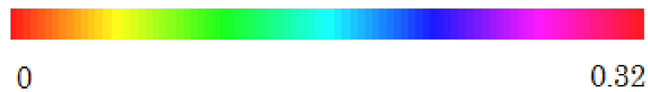
Angular distribution function of the normalized flux of radiation $\Phi_s^{(1)}(\cos(\theta),\varphi)/P_h$, backscattered by a layer of optical thickness $H = 5$. Aerozol scattering. $\theta_0 = 45°$, $\varphi_0 = 0$. Hemispherical basis.



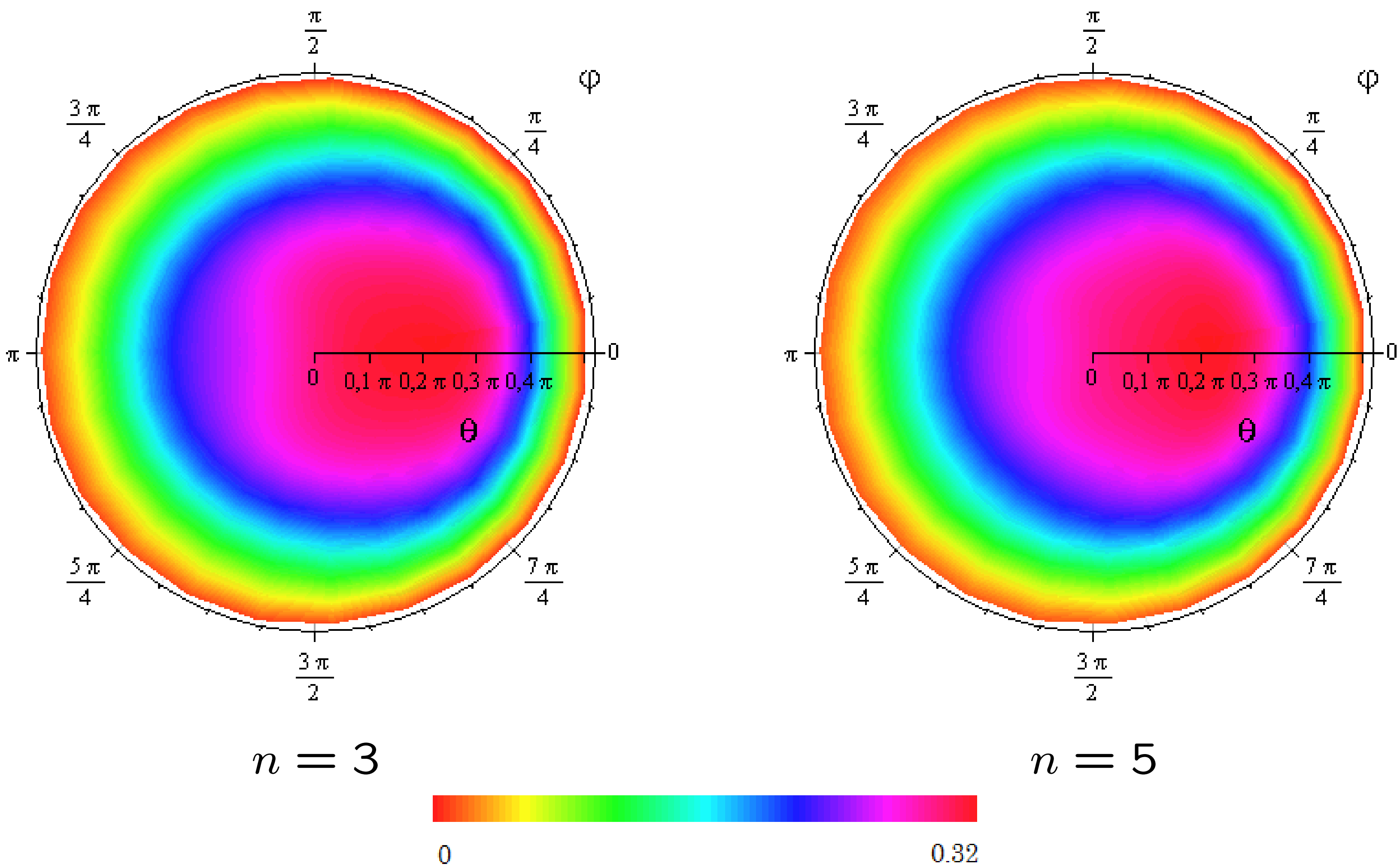
Lambertian approximation



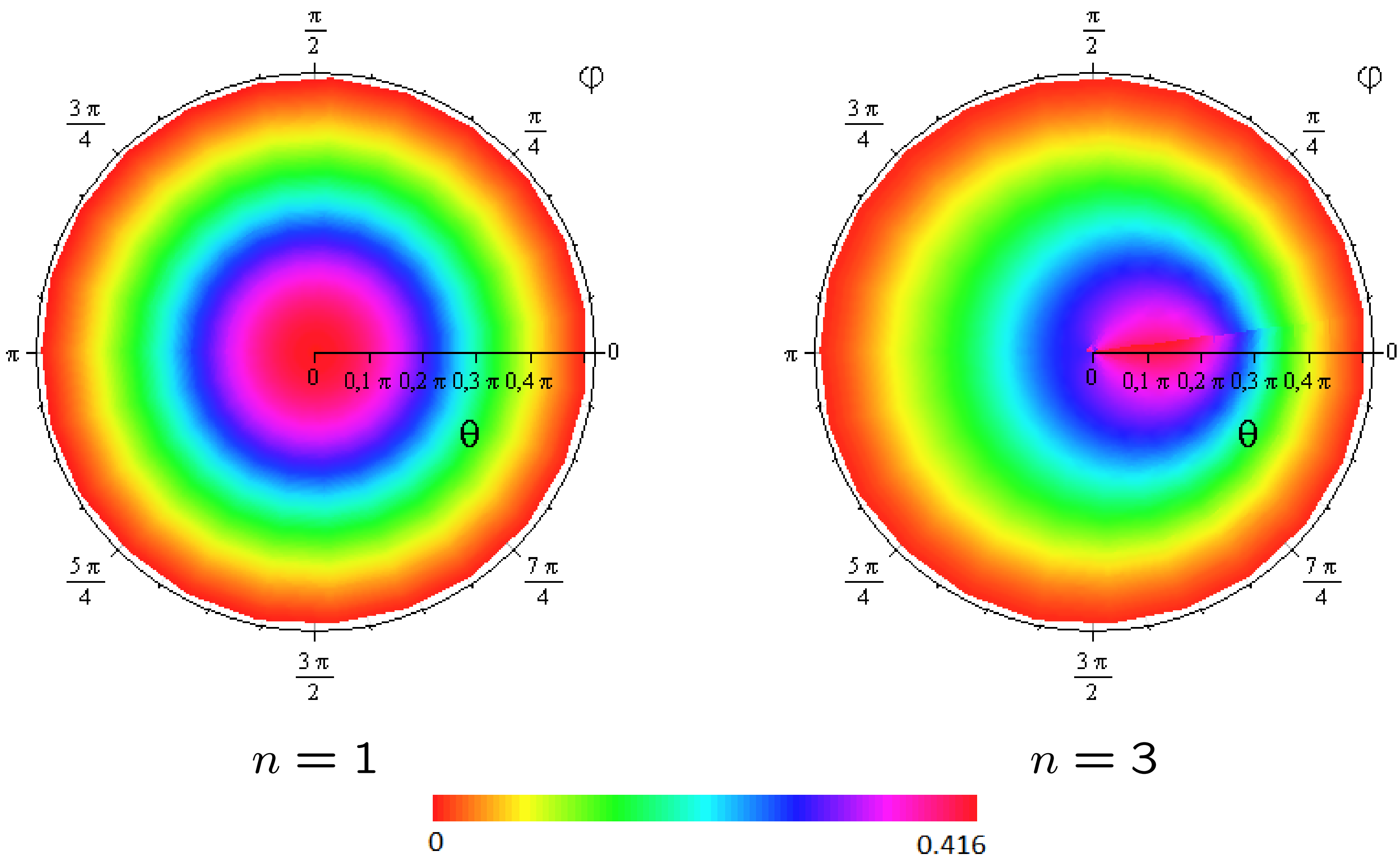
$n = 2$



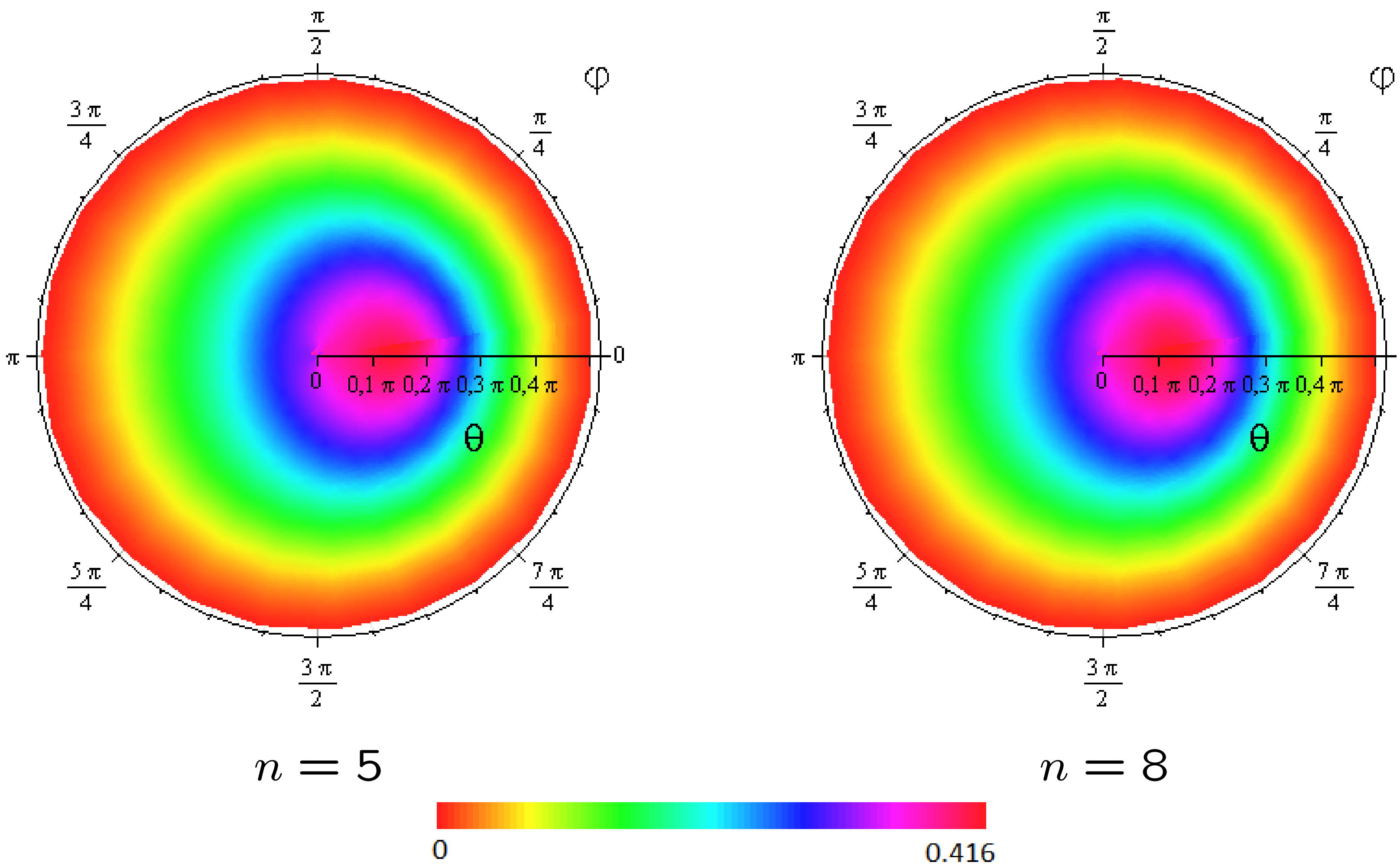
Angular distribution function of the normalized flux of radiation $\Phi_s^{(1)}(\cos(\theta), \varphi)/P_h$, backscattered by a layer of optical thickness $H = 5$. Aerozol scattering. $\theta_0 = 45°$, $\varphi_0 = 0$. Hemispherical basis.



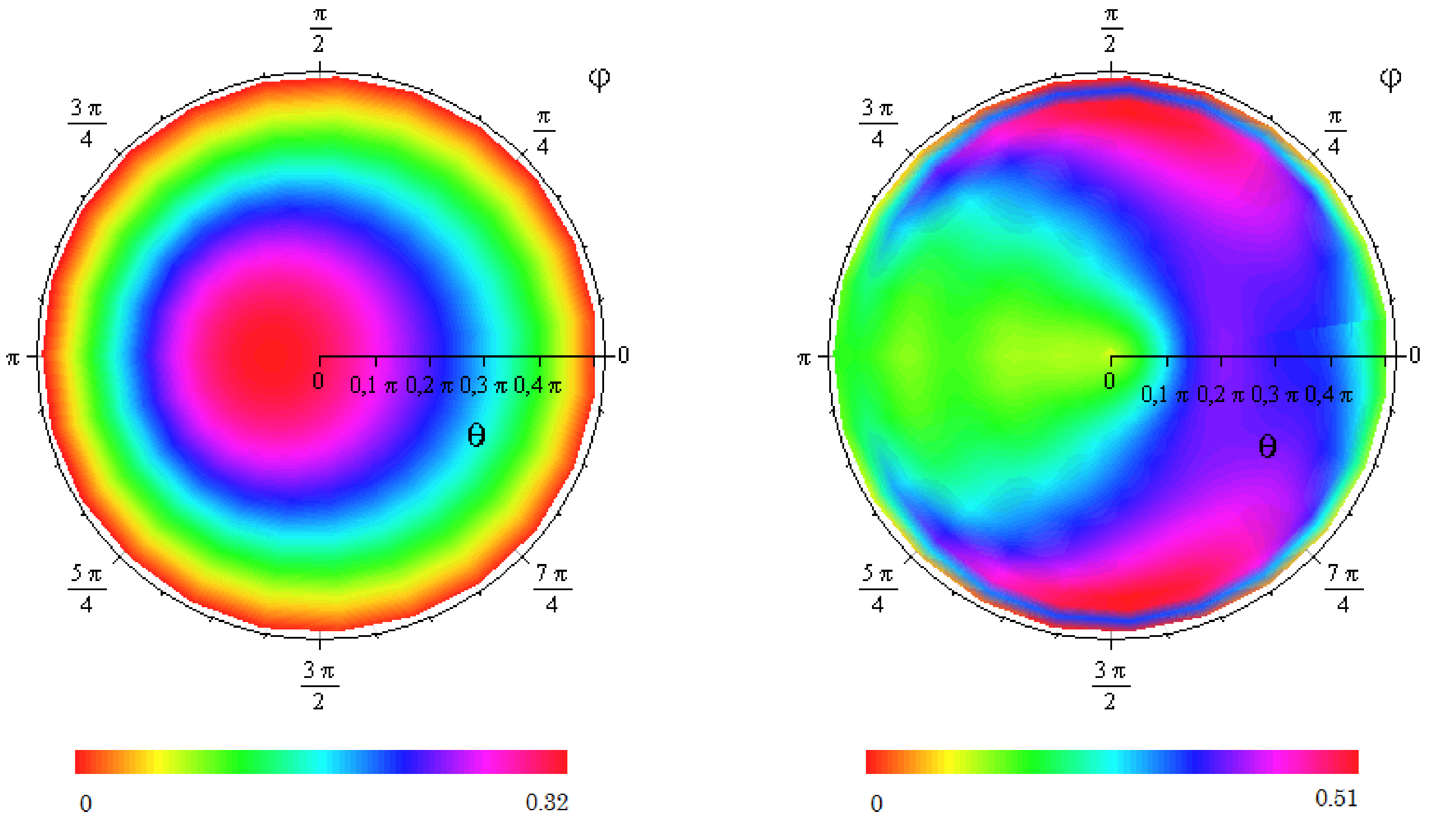
Angular distribution function of the normalized flux of radiation $\Phi_s^{(1)}(cos(\theta), \varphi)/P_h$, transmitted by a layer of optical thickness $H = 5$. Aerozol scattering. $\theta_0 = 45°$, $\varphi_0 = 0$. Factorized basis, $n_i = n_j = n$.



Angular distribution function of the normalized flux of radiation $\Phi_s^{(1)}(cos(\theta), \varphi)/P_h$, transmitted by a layer of optical thickness $H = 5$. Aerozol scattering. $\theta_0 = 45°$, $\varphi_0 = 0$. Factorized basis, $n_i = n_j = n$.

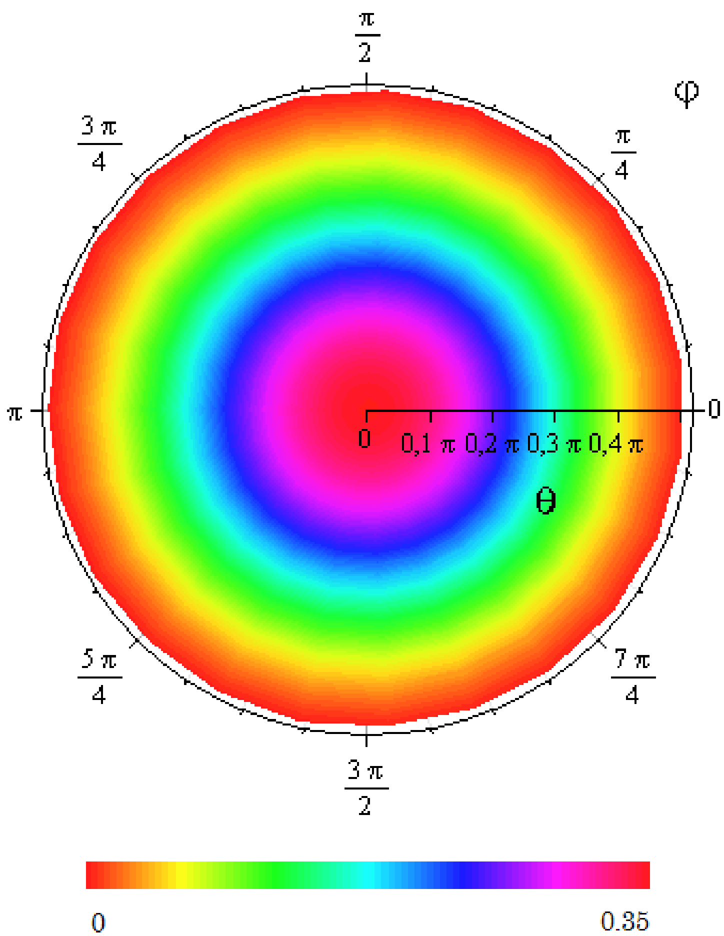


Angular distribution functions of the normalized flux of radiation $\Phi_s^{(1)}(cos(\theta),\varphi)/P_h$ and the degree of polarization $p(cos(\theta),\varphi)$, backscattered (1-st row) and transmitted (2-d row) by a layer of optical thickness $H = 5$. Molecular scattering. $\theta_0 = 45°$, $\varphi_0 = 0$. Hemispherical basis, $n = 5$.

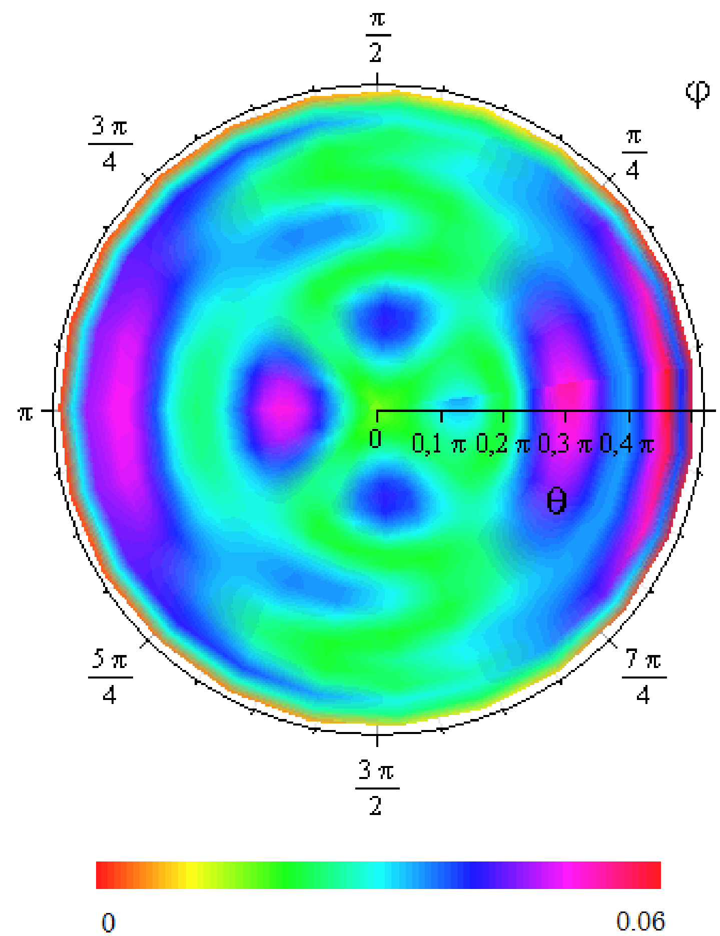


$\Phi_s^{(1)}(cos(\theta),\varphi)/P_h$ $p(cos(\theta),\varphi)$

Angular distribution functions of the normalized flux of radiation $\Phi_s^{(1)}(cos(\theta),\varphi)/P_h$ and the degree of polarization $p(cos(\theta),\varphi)$, backscattered (1-st row) and transmitted (2-d row) by a layer of optical thickness $H = 5$. Molecular scattering. $\theta_0 = 45^\circ$, $\varphi_0 = 0$. Hemispherical basis, $n = 5$.

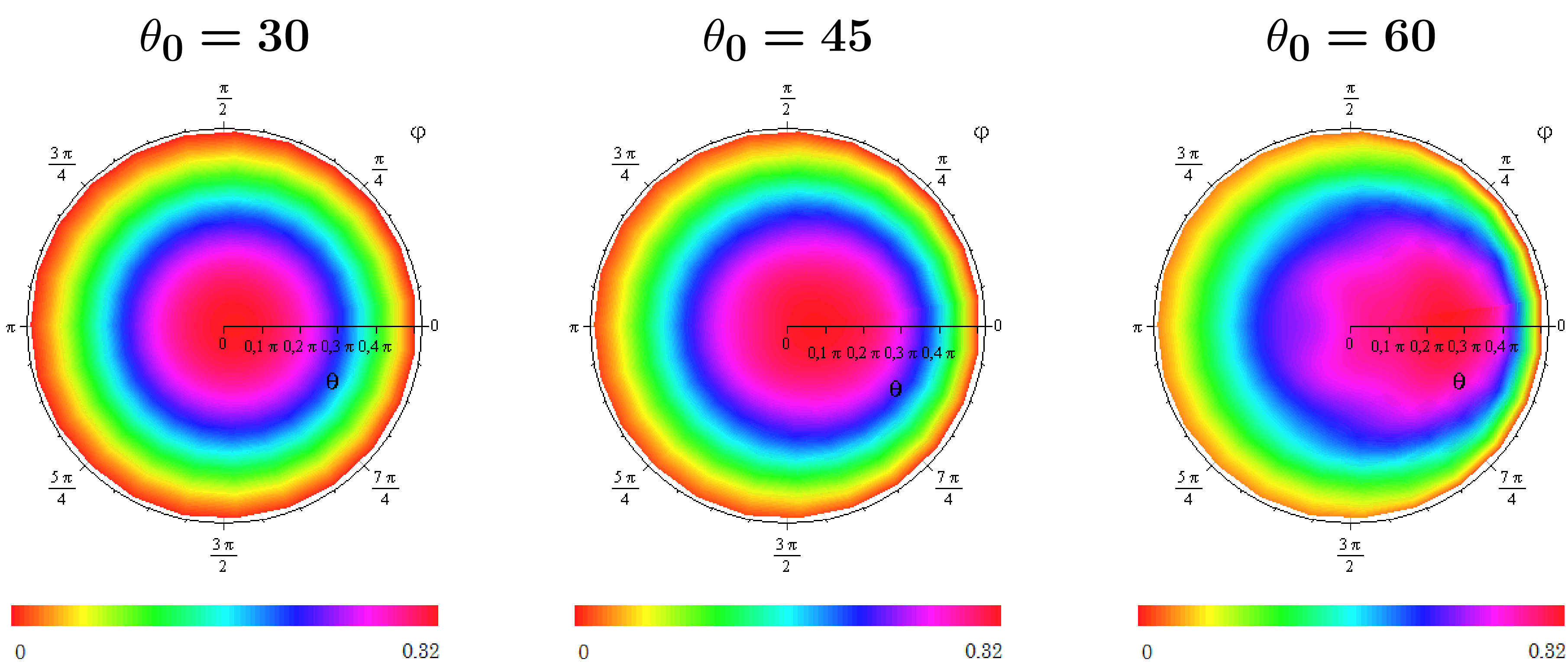


$\Phi_s^{(1)}(cos(\theta),\varphi)/P_h$

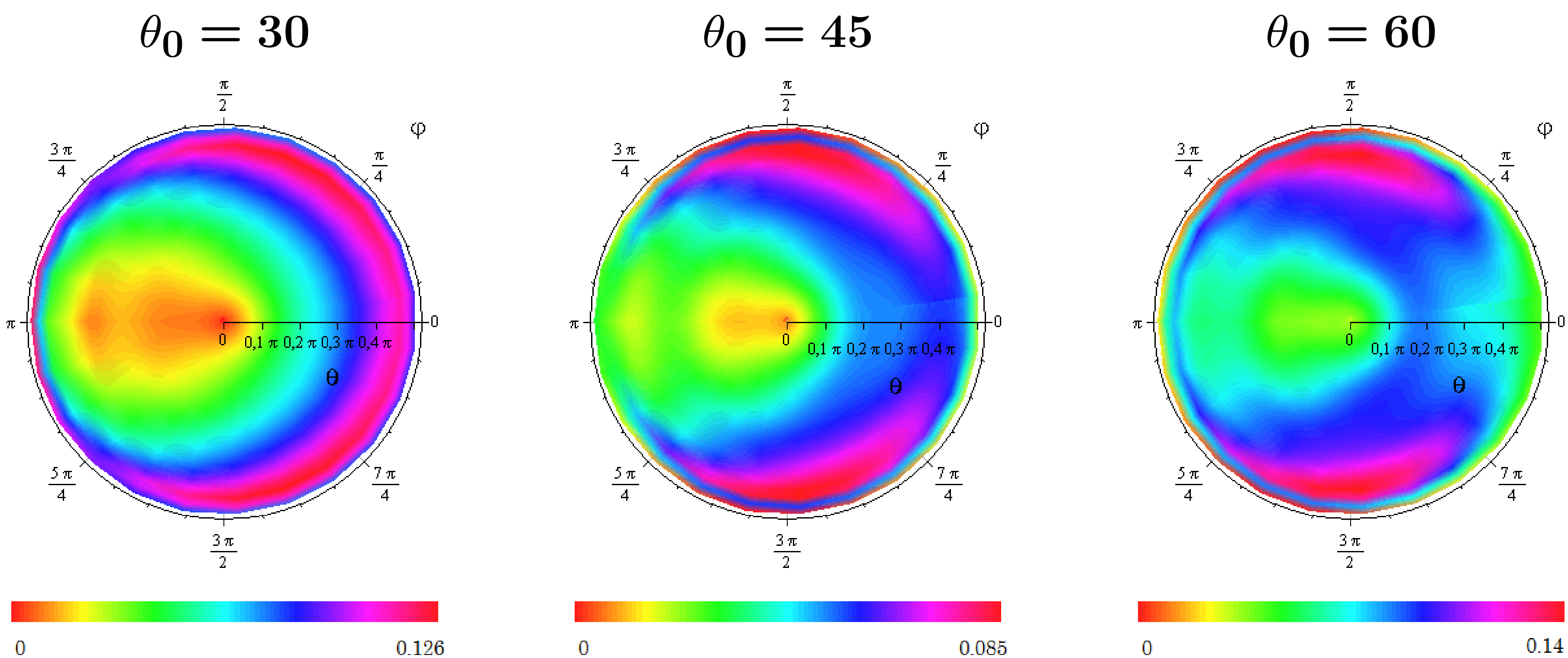


$p(cos(\theta),\varphi)$

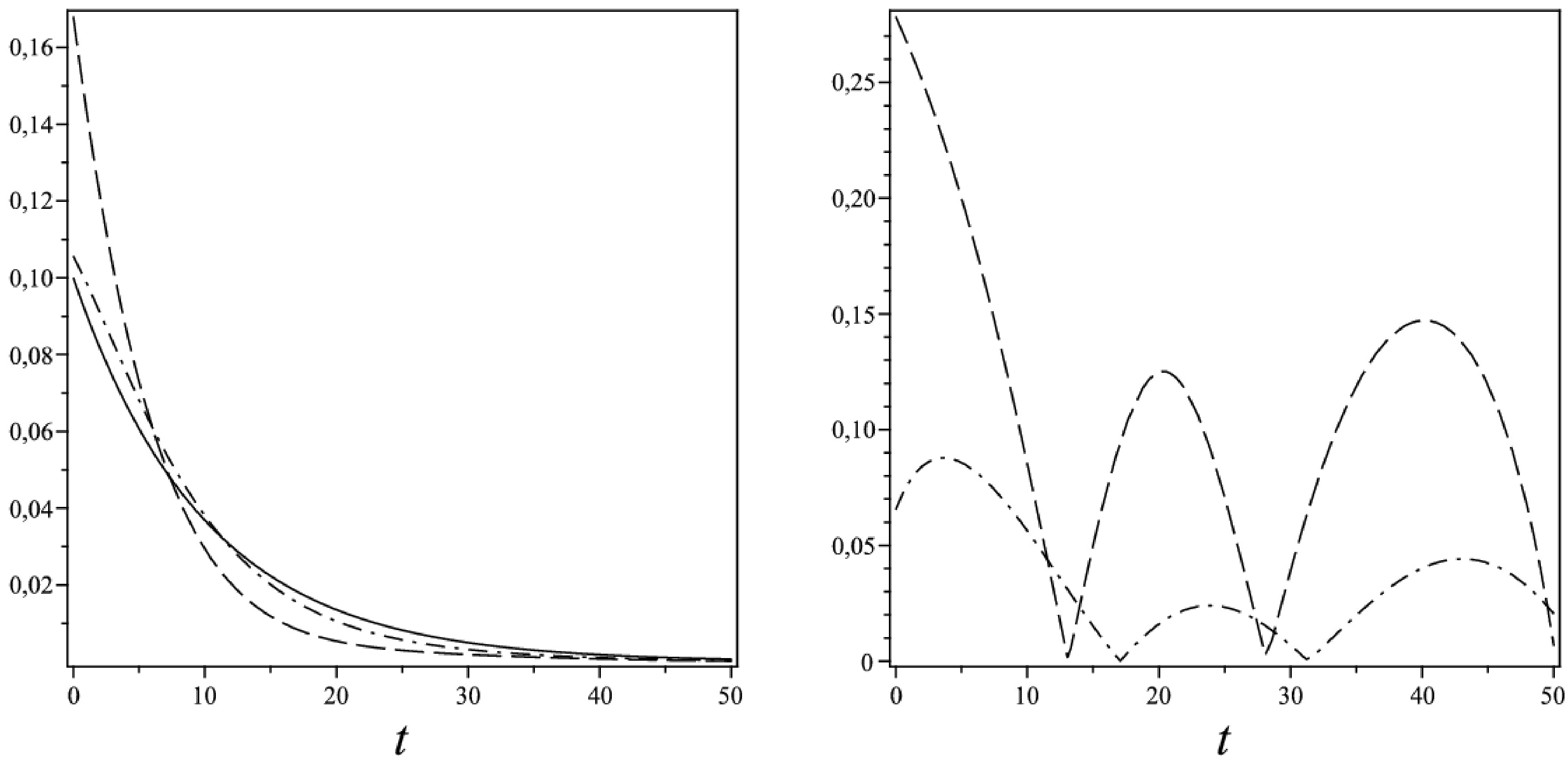
Angular characteristics of the radiation, backscattered by the layer of optical thickness $H = 10$. (a) – (c) The normalized radiation flux $\Phi_s^{(1)}(\cos(\theta), \varphi)/P_h$. (d) – (f) The degree of polarization $p_n(\cos(\theta), \varphi)$. $\varphi_0 = 0$. Hemispherical basis.



Angular characteristics of the radiation, backscattered by the layer of optical thickness $H = 10$. (a) – (c) The normalized radiation flux $\Phi_s^{(1)}(\cos(\theta), \varphi)/P_h$. (d) – (f) The degree of polarization $p_n(\cos(\theta), \varphi)$. $\varphi_0 = 0$. Hemispherical basis.

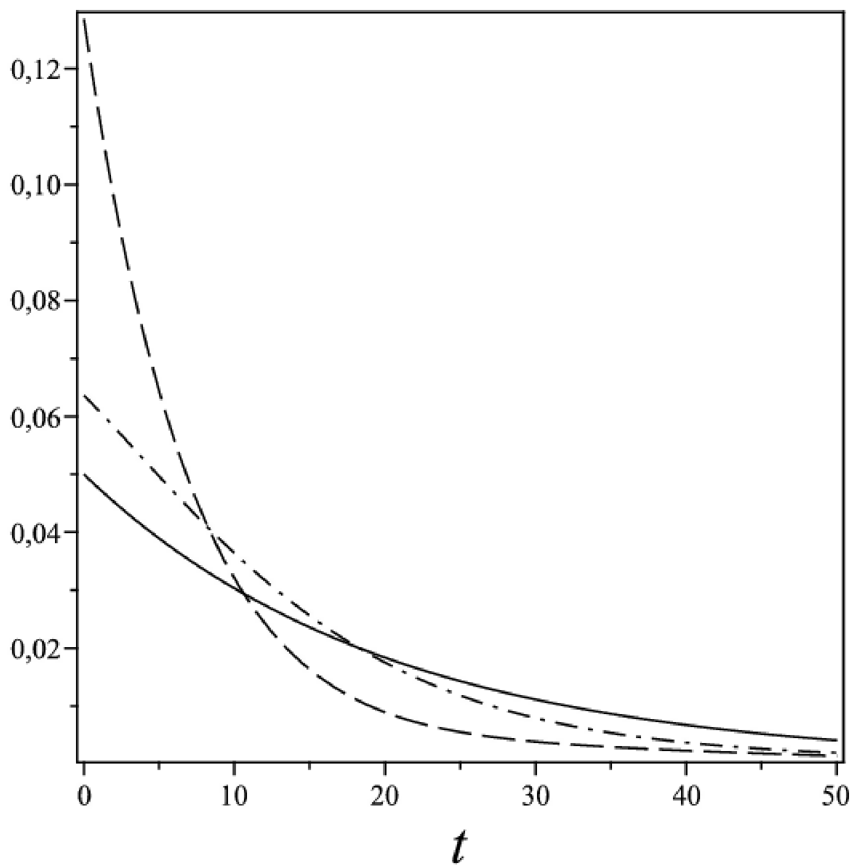


$\sigma_c = 0.1$. Solid line is for an exponential curve, dashed line is for molecular scattering matrix curve, dash-and-dot line is for aerosol scattering matrix curve. (a) Time distribution of radiation intensity, normalized over total flux, in comparison to an exponential asymptotics $e^{-\sigma_c v t}$. (b) Degree of polarization.

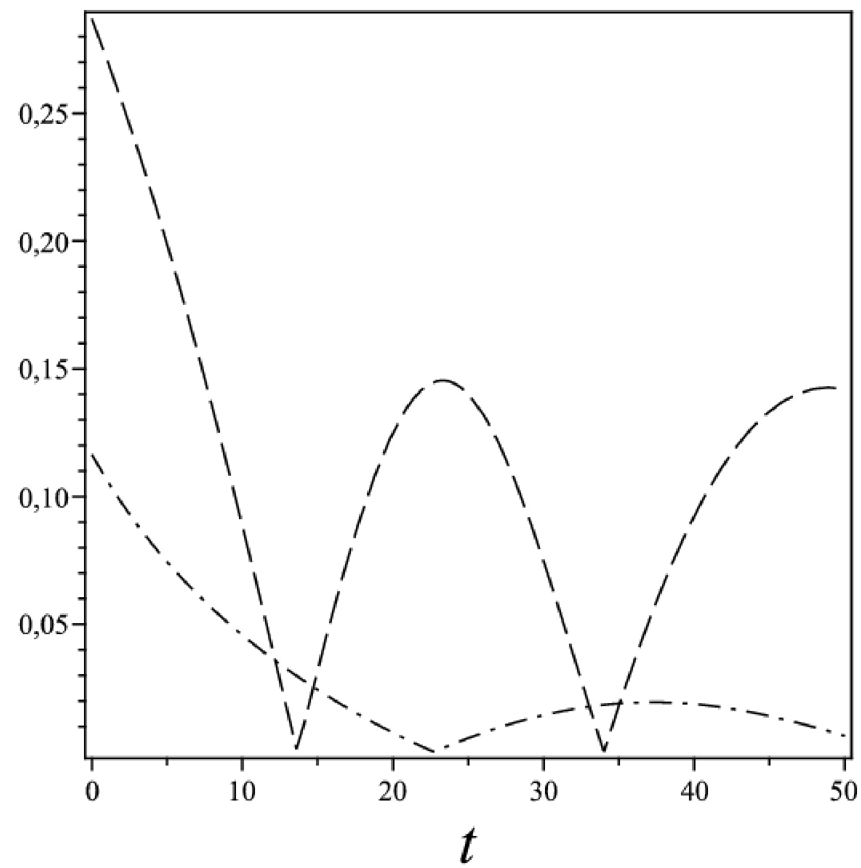


(a) (b)

$\sigma_c = 0.05$. Solid line is for an exponential curve, dashed line is for molecular scattering matrix curve, dash-and-dot line is for aerosol scattering matrix curve. (a) Time distribution of radiation intensity, normalized over total flux, in comparison to an exponential asymptotics $e^{-\sigma_c v t}$. (b) Degree of polarization.



(a)



(b)

Thank you for attention!